Modeling and Forecasting the Conditional Covariance Matrix between Stock and Bond Returns Using a Multivariate High-Frequency-Based Volatility (HEAVY) Model

Yu-Sheng Lai*

Department of Banking and Finance, National Chi Nan University, Taiwan.

Abstract

We model the dynamics of stock-bond returns using a multivariate high-frequency-based volatility (HEAVY) model. Unlike conventional GARCH models that only carry daily information, the HEAVY effectively extracting the information from high-frequency data reveals a relatively short response time and exhibits short-run momentum effects in modeling the covariance dynamics. The performance of considering both of the distinguishing properties are investigated out-of-sample using data on the S&P 500 index and 30-year T-bond assets. In an asset allocation perspective, we find that forecasts of the covariance matrix from HEAVY are significantly superior to those based on GARCH over shorter horizons of up to one week. Investors with higher risk aversions are willing to pay substantial performance fees to obtain the economic value of volatility timing using the HEAVY strategy instead of a GARCH strategy.

Keywords: realized covariance, covariance forecasts, volatility timing, asset allocation, economic value.

---

* Please address all correspondence to: Yu-Sheng Lai, Department of Banking and Finance, 1, University Road, Puli, Nantou Hsien, National Chi Nan University, 54561, Taiwan. E-mail address: yushenglai@ncnu.edu.tw. Tel: +886 49 2910960x4696. Fax: +886 49 2914511. The author is grateful to the National Science Council of Taiwan for the financial support provided for this study (NSC 102-2410-H-260-008).

1. Introduction

Stocks and bonds are two primary asset classes in investing practices, and understanding the comovement between stock and bond returns has important implications in portfolio optimization. Over years, a variety of econometric models has been employed for modeling the joint distribution of stock and bond returns.[1] Using a multivariate generalized autoregressive conditional heteroskedasticity (GARCH) model, Bollerslev et al. (1988) find that the conditional covariance matrix on stock-bond assets displays a very substantial time variation. De Goeij and Marquering (2004, 2009) find evidence for asymmetries in the conditional variances and covariance. Connolly et al. (2005) and Guidolin and Timmermann (2006) consider regime switching models for the joint distribution of US stock and bond returns. Engle and Colacito (2006) value the time variation in stock-bond correlation for asset allocation. Chou and Liu (2010) and Wu and Liang (2011) further model the conditional dependence using copula-GARCH models. These studies support the importance of understanding the relations between the volatilities and co-volatilities for spot and bond returns.

Multivariate GARCH models are widely used in many financial applications, because multi-step forecasts of covariance matrices are relatively easily obtained using the recursive generating structure. Standard GARCH models specify the covariance matrix as functions of past low-frequency (LF, hereafter) data (i.e., daily returns).[2] Andersen et

[1] It should be noted that, instead of using GARCH modelled as functions of past returns, there are studies considering certain economic factors driving the stock-bond comovement; see, e.g., Baele et al. (2010).

[2] According to the survey article of Bauwens et al. (2006), multivariate GARCH models can be divided into three; (i) generalizations of the univariate standard GARCH model; (ii) linear combinations of univariate GARCH models; (iii) nonlinear combinations of univariate GARCH models.

al. (2001) and Barndorff-Nielsen and Shephard (2002) show that latent volatility can be computed accurately using high-frequency (HF, hereafter) data (e.g., intraday returns), whereas LF returns only provide very weak signal about volatility measurements. Today many financial data sets include HF data in addition to daily prices. It is shown that incorporating realized volatility measures when modeling the dynamic properties of volatility is very beneficial, and leads to better empirical fit than standard GARCH that only use LF returns (Engle, 2002; Shephard and Sheppard, 2010; Hansen et al., 2012).

In line with the recently studies, this paper considers a multivariate volatility model with HF data to specify the dynamics between stock and bond returns. The multivariate high-frequency-based volatility (HEAVY) model introduced by Noureldin et al. (2012) is adopted to model the conditional covariance matrix dynamics. Compared with the standard GARCH, covariance forecasts from HEAVY model have a relative short response time and exhibit short-run momentum effects. The former indicates that the HEAVY responses quickly during times of rapid changes in volatility and correlation. The latter says that the tendency of volatility forecasts can continue over short horizons before mean reverting. By effectively extracting information about the current levels of volatilities and correlations from HF data, the HEAVY model shall be particularly useful for modeling stock-bond returns during periods of rapid changes in the underlying covariance structure.

Portfolio optimization is a natural application for evaluating the forecasts from HEAVY model. In the classical asset allocation framework, an investor has to choose portfolio weights to minimum portfolio variance subject to a required return constraint. Accordingly, investors with different covariance forecasts will hold different portfolios. To avoid estimating the true mean returns as discussed by Jagannathan and Ma (2003), we measure the value of covariance information by finding the global minimum variance portfolios (GMVP) using US stock and bond data over a 4-year period covering the 2008 subprime crisis. An advantage of studying GMVP is that the corresponding portfolio weights are determined only on forecasts of the covariance matrix for the given the investment horizon. This allows us to isolate the mean impacts and focus on comparing the covariance forecasts in an asset allocation perspective. In addition to conduct statistical comparisons, the benefits of covariance forecasts using HF data are assessed by adopting the conditional utility-based evaluation approach suggested by Hautsch et al. (2013).

The structure of the paper is as follows. In the next section we introduce the HEAVY model, and in Section 3 we present the GMVP framework for evaluating the performance of volatility timing. Section 4 contains the results of our empirical analysis, while Section 5 concludes the paper.

2. The Bivariate HEAVY Model

This section introduces a new class of volatility model that utilizes high-frequency data to describe the co-movement between stock and bond returns. We employ the multivariate HEAVY model of Noureldin et al. (2012) to model the conditional covariance matrix. Unlike standard GARCH models, covariance forecasts from HEAVY model have a relative short response time and exhibit short-run momentum effects. The distinguishing properties might have essential advantages in performing volatility-timing strategies. The details of the model are as follows.

Let $\mathbf{R}_t$ denote an 2×1 vector of daily returns consisting of stock and bond assets. Given the success of GARCH models, we assume that the return vector can be decomposed as follows:

$$\mathbf{R}_t = \mu_t + \mathbf{H}_t^{1/2} \mathbf{z}_t , \tag{1}$$

where $\mu_t := \mathrm{E}[\mathbf{R}_t \mid \mathcal{F}_{t-1}]$ and $\mathbf{H}_t := \mathrm{E}[(\mathbf{R}_t - \mathbf{m}_t)^2 \mid \mathcal{F}_{t-1}]$, respectively, represent the conditional mean vector and the conditional covariance matrix, conditioned on the information set $\mathcal{F}_{t-1}$. Furthermore, we assume that the random vector $\mathbf{z}_t$ satisfies $\mathrm{E}[\mathbf{z}_t] = 0$ and $\mathrm{var}[\mathbf{z}_t] = \mathbf{I}_2$, where $\mathbf{I}_2$ is an identity matrix of order 2. To keep simplicity, the conditional means are assumed time-invariant, though a vector autoregressive (VAR) representation could be adopted as in, e.g., De Goeij and Marquering (2009).

What remains to be specified is the covariance matrix process $\mathbf{H}_t$. Over years, various parametric GARCH-type models have been employed for this purpose. The essential characteristic of these models is that they utilize lagged LF data for the construction of $\mathbf{H}_t$. Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004) indicate that returns yield very weak signals about latent volatility, whereas HF-based realized measures provide very accurate estimates. This implies that these standard GARCH models might perform poorly in situations where volatility/correlation changes rapidly to a new level (Noureldin et al., 2012). Accordingly, GARCH-like models encompassing finer realized measures are recently introduced; see Engle (2002), Fleming et al. (2003), Hansen et al. (2014), and Noureldin et al. (2012) for examples. The coincidence is that, volatility forecasts use lagged realized measures are superiority to those use lagged squared/cross-product returns.

We model the covariance matrix by the recently introduced multivariate HEAVY model of Noureldin et al. (2012). Denote $\mathbf{V}_t$ the 2×2 realized measure and $\mathbf{P}_t = \mathbf{u}_t \mathbf{u}_t'$ the 2×2 outer product of daily returns, where $\mathbf{u}_t := \mathbf{H}_t^{1/2} \mathbf{z}_t$ given in (1). The two-equation model approached by the BEKK parameterization of Engle and Kroner (1995) can be compactly written as

$$\mathrm{E}[\mathbf{P}_t \mid \mathcal{F}_{t-1}^{\mathrm{HF}}] \equiv \mathbf{H}_t = \bar{\mathbf{C}}_H \bar{\mathbf{C}}_H' + \bar{\mathbf{B}}_H \mathbf{H}_{t-1} \bar{\mathbf{B}}_H' + \bar{\mathbf{A}}_H \mathbf{V}_{t-1} \bar{\mathbf{A}}_H' \tag{2}$$

$$\mathrm{E}[\mathbf{V}_t \mid \mathcal{F}_{t-1}^{\mathrm{HF}}] \equiv \mathbf{M}_t = \bar{\mathbf{C}}_M \bar{\mathbf{C}}_M' + \bar{\mathbf{B}}_M \mathbf{M}_{t-1} \bar{\mathbf{B}}_M' + \bar{\mathbf{A}}_M \mathbf{V}_{t-1} \bar{\mathbf{A}}_M', \tag{3}$$

where $\bar{\mathbf{C}}_H$ and $\bar{\mathbf{C}}_M$ are 2×2 lower-triangular parameter matrices, $\bar{\mathbf{B}}_H$, $\bar{\mathbf{A}}_H$, $\bar{\mathbf{B}}_M$ and $\bar{\mathbf{A}}_M$ are 2×2 parameter matrices. The parameterization guarantees the positive definite of $\mathbf{H}_t$ for all t under mild restrictions. In the case of a diagonal HEAVY model, $\bar{\mathbf{B}}_H$, $\bar{\mathbf{A}}_H$, $\bar{\mathbf{B}}_M$ and $\bar{\mathbf{A}}_M$ each have 2 free parameters, and $\bar{\mathbf{C}}_H$ and $\bar{\mathbf{C}}_M$ each have 3 free parameters.
Noureldin et al. (2012) call (2) the HEAVY-P equation, where (3) is named the HEAVY-V equation. The latter is necessary for multi-step forecasts of $\mathbf{H}_t$ due to the fact that $\mathbf{V}_{t-1}$ is present in (2).

The HEAVY system utilizes recently developed estimators of quadratic covariation that have shown to be more precise compared to outer product of daily returns. For example, a realized covariance matrix estimator on day t is defined as $\mathbf{V}_t = \sum_{j=1}^{n} r_{j,t} r_{j,t}'$, where $r_{j,t}$ denote the jth uniformed spaced vector of returns. In the presence of microstructure noise, one shall switch to some noise-robust realized covariance estimators for a correction purpose. Accordingly, this paper uses the realized kernel (RK) of Barndorff-Nielsen et al. (2011) for the construction of $\mathbf{V}_t$, as follows:

$$\mathrm{RK} = \sum_{h=-n}^{n} k\left(\frac{h}{L}\right)\Gamma_h, \tag{4}$$

where $\Gamma_h = \sum_{j=h+1}^{n} x_j x'_{j-h}$ for $h >= 0$ and $\Gamma_h = \Gamma'_{-h}$ for $h < 0$, x_j is the high frequency vector returns defined by jittering end conditions, L is the bandwidth parameter aims to control the number of leads and lags used in the non-stochastic Parzen kernel function $k(\cdot)$. Under a general form of noise, the authors show that the RK is consistent and positively definite for the latent covariance matrix.

Incorporating realized measures when modeling the dynamic properties of volatility is very beneficial, and leads to better empirical fit than the conventional GARCH model

$$\mathrm{E}[\mathbf{P}_t \mid \mathcal{F}_{t-1}^{\mathrm{LF}}] := \mathbf{H}_t = \overline{\mathbf{C}}_H \overline{\mathbf{C}}'_H + \overline{\mathbf{B}}_H \mathbf{H}_{t-1} \overline{\mathbf{B}}'_H + \overline{\mathbf{A}}_H \mathbf{P}_{t-1} \overline{\mathbf{A}}'_H, \tag{5}$$

which only uses daily returns. Noureldin et al. (2012) indicate that the primary distinction between HEAVY and GARCH is the conditioning information set $\mathcal{F}_{t-1}$. HEAVY conditions on $\mathcal{F}_{t-1}^{\mathrm{HF}}$ influenced by past realized volatility measures, whereas GARCH conditions on $\mathcal{F}_{t-1}^{\mathrm{LF}}$ influenced by past daily returns. They show that HEAVY models have a relatively short time response than that of standard GARCH models, meaning the formers' forecasts are faster in situations where the level of volatility or correlation is subject to abrupt changes.

Estimations of (2) and (3) can be carried out equation-by-equation via quasi-maximum likelihood (QML) estimators. Let ϑ_H and ϑ_M respectively denote the true parameter vectors of HEAVY-P and HEAVY-V equations with the following log-likelihood functions:

$l_{H,t}(\vartheta_H) = c_H - \frac{1}{2}\left(\log\left|\mathbf{H}_t\right| + tr(\mathbf{H}_t^{-1}\mathbf{P}_t)\right)$ and

$l_{M,t}(\vartheta_M) = c_M - \frac{n}{2}\left(\log\left|\mathbf{M}_t\right| + tr(\mathbf{M}_t^{-1}\mathbf{V}_t)\right)$, where c_H and c_M are some constants. By maximizing the log-likelihood functions, we obtain the QML estimators as

$$\hat{\vartheta}_H = \arg\max_{\vartheta_H} \sum_{t=1}^{T} l_{H,t}(\vartheta_H) \quad \text{and} \quad \hat{\vartheta}_M = \arg\max_{\vartheta_M} \sum_{t=1}^{T} l_{M,t}(\vartheta_M), \tag{6}$$

where T stands for the total number of observations. By imposing a strictly stationary and ergodic solution, the QML estimators show strong consistency for the HEAVY system. The covariance stationary condition given in Noureldin et al. (2012) is analogous to the one in Engle and Kroner (1995).

Forecasting the conditional covariance matrix of daily returns is a key input for volatility timing (Fleming et al., 2001, 2003). In the HEAVY system, one-step forecasts are directly computable using (2). When s-step forecasts of $\mathbf{H}_t$ are needed, the proposition 2 in Noureldin et al. (2012) gives the s-step forecasts of $\mathbf{H}_t$ for $s = 2, 3, \ldots$, can be expressed as

$$\begin{aligned} \mathrm{E}[h_{t+s} \mid \mathcal{F}_t^{\mathrm{HF}}] &= \sum_{i=1}^{s-1} B_H^{i-1} C_H + B_H^{s-1} h_{t+1} \\ &\quad + \sum_{i=1}^{s-1} B_H^{i-1} A_H \left\{ \sum_{j=1}^{s-i-1} (B_M + A_M)^{j-1} C_M + (B_M + A_M)^{s-i-1} m_{t+1} \right\}, \end{aligned} \tag{7}$$

where $h_t := \mathrm{vech}(\mathbf{H}_t)$ and $m_t := \mathrm{vech}(\mathbf{M}_t)$ are 3×1 vectors by stacking the lower triangular part and the main diagonal of the 2×2 matrices, $C_H = L_2(\overline{\mathbf{C}}_H \otimes \overline{\mathbf{C}}_H) D_2 vech(I_2)$, $B_H = L_2(\overline{\mathbf{B}}_H \otimes \overline{\mathbf{B}}_H) D_2$ and $A_H = L_2(\overline{\mathbf{A}}_H \otimes \overline{\mathbf{A}}_H) D_2$ are redefined using the non-stochastic elimination and duplication matrices L_2 and D_2. Noureldin et al. (2012) exhibit that the profile of forecasts from HEAVY models differs from GARCH models particularly in persistence and short-run momentum effects. The momentum effects say that, before mean reverting, the tendency of volatility forecasts can continue over short horizons, whereas the GARCH forecasts by (5) only reveals monotonically mean reverts. The differences point to the information content of realized measures for the forecasts of $\mathbf{H}_t$.

3. Evaluations by Finding the Global Minimum Variance Portfolios

In this section, we describe the methodology for measuring the economic value of volatility timing. We examine the economic gains of constructing the GMVP in terms of forecasts of the conditional covariance matrix. The GMVP problem can be formulated as

$$\min_{\mathbf{w}_{t,t+h}} \mathbf{w}'_{t,t+h}\Sigma_{t,t+h}\mathbf{w}_{t,t+h} \quad \text{subject to} \quad \mathbf{w}'_{t,t+h}\iota = 1, \tag{8}$$

where $\mathbf{w}_{t,t+h}$ is the 2×1 vector of portfolio weights, $\Sigma_{t,t+h}:=\mathrm{cov}[\mathbf{R}_{t,t+h} \mid \mathcal{F}_t]$ is the 2×2 true covariance matrix of returns $\mathbf{R}_{t,t+h}$ from day t to $t+h$, and ι is a 2×1 vector of ones. For simplicity, we assume $\Sigma_{t,t+h}=\sum_{r=1}^{h}\mathrm{E}[\Sigma_{t+r-1,t+r}|\mathcal{F}_t]$. Solving the problem yields the GMVP portfolio weights given by

$$\tilde{\mathbf{w}}_{t,t+h} = \frac{\Sigma^{-1}_{t,t+h}\iota}{\iota'\Sigma^{-1}_{t,t+h}\iota}, \tag{9}$$

and the corresponding portfolio return and conditional portfolio variance are given by $\tilde{\mathbf{w}}'_{t,t+h}\mathbf{R}_{t,t+h}$ and $\tilde{\mathbf{w}}'_{t,t+h}\Sigma_{t,t+h}\tilde{\mathbf{w}}_{t,t+h}$, respectively.

The general mean-variance portfolio optimization problem involves a vector of expected excess returns (over the risk-free asset) on the risky assets. To avoid the effect of estimation error in the mean vector on portfolio weights, recent studies suggest investing into the GMVP instead of the tangency portfolio (e.g., DeMiguel et al., 2009; Hautsch et al., 2013;

Jagannathan and Ma, 2003). The GMVP portfolio weights are a special case when restricting the expected returns are identical and to be ones across all assets.

To compare the performance of covariance forecasts, Patton and Sheppard (2009) illustrate a basic rule within the GMVP framework. Let $\mathbf{H}_{t,t+h} := \sum_{r=1}^{h} \mathbf{H}_{t,t+r}$ for $h \geq 1$ be an estimated covariance matrix from day t to $t+h$, and $\hat{\mathbf{w}}_{t,t+h}$ be the associated GMVP weights computed according to (9). Then the conditional variance of GMVP based on $\hat{\mathbf{w}}_{t,t+h}$ must be larger than that based on the true portfolio weights $\tilde{\mathbf{w}}_{t,t+h}$:

$$\tilde{\mathbf{w}}'_{t,t+h} \Sigma_{t,t+h} \tilde{\mathbf{w}}_{t,t+h} < \hat{\mathbf{w}}'_{t,t+h} \Sigma_{t,t+h} \hat{\mathbf{w}}_{t,t+h}. \tag{10}$$

The implication is that the lower variance bound can only be attained if we know the true covariance matrix of the asset returns. To compute the conditional portfolio variance in practice, Hautsch et al. (2013) proxy $\Sigma_{t,t+h}$ in (10) by the realized covariance matrix $\mathrm{Rcov}_{t,t+h}$ form day t to $t+h$, and obtain the resulting conditional variance estimate by

$$\sigma^{2,p}_{t,t+h} := \hat{\mathbf{w}}'_{t,t+h} \mathrm{Rcov}_{t,t+h} \hat{\mathbf{w}}_{t,t+h}. \tag{11}$$

Using this result, we can compare two competing covariance forecasts by comparing the conditional variance of the GMVP constructed by the models.

In addition to statistical evaluations, the benefits of covariance forecasts are assessed by adopting the conditional utility-based evaluation approach suggested by Hautsch et al. (2013). Assume that the risk-averse investor is endowed with the quadratic utility function

$$U(r^p_{t,t+h}) = 1 + r^p_{t,t+h} - \frac{\gamma}{2(1+\gamma)}(1 + r^p_{t,t+h})^2, \tag{12}$$

where $r^{p}_{t,t+h} := \hat{\mathbf{w}}'_{t,t+h}\mathbf{R}_{t,t+h}$ is the portfolio return associated with the estimated GMVP weights, $\gamma = 1,10$ denote the investor's relative risk aversions that follow Fleming et al. (2003) and DeMiguel et al. (2009). Given the portfolio returns $r^{p,\mathrm{GARCH}}_{t,t+h}$ and $r^{p,\mathrm{HEAVY}}_{t,t+h}$, Δ_{γ} represents a fee the investor is willing to pay to switch from GARCH to HEAVY forecasts, given by

$$\sum_{t=1}^{T-h} \mathrm{E}[U(r^{p,\mathrm{GARCH}}_{t,t+h}) \mid \mathcal{F}_{t}] = \sum_{t=1}^{T-h} \mathrm{E}[U(r^{p,\mathrm{HEAVY}}_{t,t+h} - \Delta_{\gamma}) \mid \mathcal{F}_{t}]. \tag{13}$$

Hautsch et al. (2013) indicate that the solution Δ_{γ} in (13) depends on the conditional portfolio variances, $\hat{\mathbf{w}}'_{t,t+h}\Sigma_{t,t+h}\ \hat{\mathbf{w}}_{t,t+h}$, and the conditional means, $\hat{\mathbf{w}}'_{t,t+h}\mu_{t,t+h}$. Given that the expected returns $\mu_{t,t+h} = (h / 252)\mu^{\mathrm{id}}\iota$ are constant over time and identical across all assets for $t = 1,\ldots,T-h$, the analytical solution for Δ_{γ} is given by

$$\Delta_{\gamma} = \frac{h\mu^{\mathrm{id}}}{252} - \frac{1}{\gamma} + \sqrt{\left(\frac{h\mu^{\mathrm{id}}}{252} - \frac{1}{\gamma}\right)^{2} + \overline{\sigma^{2,p}_{\mathrm{GARCH}}} - \overline{\sigma^{2,p}_{\mathrm{HEAVY}}}}, \tag{14}$$

where

$$\overline{\sigma^{2,p}} := \frac{1}{T-h}\sum_{t=1}^{T-h} \hat{\mathbf{w}}'_{t,t+h}\mathrm{Rcov}_{t,t+h}\ \hat{\mathbf{w}}_{t,t+h}.$$

A grid of values of $\mu^{\mathrm{id}} \in \{-0.05, 0, 0.05, 0.1\}$ are suggested to control the impact of the assumed values on the switching fee Δ_{γ}. Under the assumption that $(h / 252)\mu^{\mathrm{id}} \leq 1/\gamma$, we obtain $\Delta_{\gamma} > 0$ when $\overline{\sigma^{2,p}_{\mathrm{GARCH}}} > \overline{\sigma^{2,p}_{\mathrm{HEAVY}}}$. This evaluation approach is in spirit of Fleming et al. (2003), but transfers the unconditional to a conditional framework. Since the relationship in (10) holds only for conditional but not for unconditional variances, Hautsch et al. (2013) indicate that gains might be understated when the evaluations are carried using unconditional variances.

4. Data Description and Empirical Results

We use HF data on S&P 500 equity index (symbol: SP) and 30-year Treasury bond (symbol: US) obtained from Tick Data Inc. to represent the stock and bond markets. The sample period is January 2, 1998 to December 31, 2007 with a total of 3528 trading days. According to Fleming et al. (2003), futures prices are used in terms of excess returns for volatility timing strategies. This also avoid the short sale constrains in asset allocation. Bonds with more than 10 years to maturity are considered in this paper, because long-term bonds are effectively matching their duration with stocks.[3]

Estimations of a HEAVY system require both the information of returns and realized measures of volatility. To construct the return series, we generally use the nearby contract in each market as in Fleming et al. (2003). This means that when the trading volume of the nearest to expiry contracts are exceeded by the front month contracts, in the following business day the nearest contracts are switched to the second nearest to maturity contracts. The trading hours for stock contracts are from 8:30 a.m. to 3:15 p.m. whereas for bond contracts are from 7:20 a.m. to 2:00 p.m. (Central Time). Our analysis focuses on modeling the joint distribution when both markets are open; therefore we assume that the GMVP positions are created immediately after 8:30 a.m. and are closed before 2:00 p.m. for the trading day. Accordingly, we measure the vector of daily

---

[3] Connolly et al. (2005) indicate that their results using the 30-year bonds are qualitatively similar to those using the 10-year bonds. Accordingly, we choose to report the results using the 30-year T-bond.

returns, $\mathbf{R}_t$, by the vector of open-to-close returns, as the logarithmic difference of closing (2:00 p.m.) and opening (8:30 a.m.) prices. This treats overnight returns as deterministic jumps as in Andersen et al. (2010) and matches the constructed open-to-close realized covariance matrices. Table 1 provides summary statistics for the returns, for both open-to-close and close-to-close. As can be seen from the numbers, there do exist considerable overnight jumps especially for the stock market. In addition, including the noisy overnight returns might also diminish the performance difference between forecasts using LF and HF data. This provides the rationality of using jump-adjusted returns instead of that without jump corrections.

＜Table 1 is inserted about here＞

For the daily realized variances and covariance, the multivariate RK estimator as in (4) are used for the construction of $\mathbf{V}_t$. Instead of using tick-by-tick data, we use 1-minute synchronized returns obtained from Tick Data Inc. for the computations. As indicated by Chaboud et al. (2010), in deep and liquid markets like 10-year Treasury notes one can sample the returns once every 30 to 40 seconds using the realized kernel. For the bandwidth choice in (9), we follow the procedure from Barndorff-Nielsen et al. (2011). Averagely, bandwidth choice during the sample period is around 14, ranging from 4 to 22. The numbers are applied to control the number of leads and lags in (9) for each business day. Under a general form of noise, it is shown that the multivariate RK with Parzen weight function is consistent and positively definite for the latent covariance matrix. In Figure 1 we graph the time series for the daily realized variances and correlations. The stock-bond return correlation displays very substantial time variations. The large negative spikes in realized correlations are either associated with deep decrease in stock market values or coincides with a deflation scare, as in Baele et al. (2010). The negative correlation at the end of 1998 or 2007 for examples ascribed to a "flight-to-safety" phenomenon by Connolly et al. (2005) induces investors to flee stocks in favor of bonds.

<Figure 1 is inserted about here>

After obtaining daily returns and realized measures, Table 2 exhibits the estimation results of the spot-futures distribution interpreted by the models. We split the full sample data in two: the period from 1998 to 2007 is regarded as in-sample for estimation of the models; and the remaining 4 years of data is used for out-of-sample analysis. We use the constrained numerical optimization procedure of MATLAB with the covariance stationary condition imposed for the estimations. In order to improve convergence, we use starting values based on the preliminary unconditional statistics with a range of rational values to ensure that the estimation procedure converges to a global maximum. The log-likelihood function with eigenvalues for each model is reported in the bottom of Table 2.

<Table 2 is inserted about here>

To test the significance of switching from GARCH to HEAVY-P, we apply the non-nested likelihood ratio test of Vuong (1989) based on the Kullback-Leibler information criterion. The test statistic is 3.33, indicating the significance of the improvement is relatively large. The eigenvalues interpreted by HEAVY-V range between 0.97 and 0.99. The values must be positive and lower than 1 to remain stationary. The smaller eigenvalues indicate a less persistence in variances and covariance, relative to the estimates obtained by GARCH. The lower estimates of b_{ii} by HEAVY indicate that a current shock does not permanently affect the conditional variance of all future horizons. Accordingly, putting higher weights on a_{ii} in HEAVY-P provides the potential for rapidly response to the changing markets. In the top panel of Figure 2 we give the level of volatility in the stock market, whereas the result for the bond is given in the middle panel. For the conditional correlations (implied by the BEKK representation), we give the figure in the bottom panel. The plots clearly reveal that GARCH provides smoothed estimates in fitting the data. Accordingly, GARCH models might perform poorly in situations where volatility changes rapidly to a new level.

＜Figure 2 is inserted about here＞

Table 3 reports the GMVP performance of HEAVY and GARCH forecasts over the forecasting horizons. As can be seen from the numbers, the covariance forecasts based on HEAVY model are superior to those based on GARCH in terms of GMVP volatility. The superiority can last up to 27 days as evidenced by the variance ratio greater than 100. The performance is more pronounced up to 7 prediction days by conducting a DMW forecast comparison test of Diebold and Mariano (1995) and West (1996, 2006). In addition to statistical evaluations, economic gains of using HEAVY model are also reported in the last two column of the table, under the assumption of $\mu^{\mathrm{id}} = 0.05$. Using the conditional utility-based approach, the results clearly indicate that the switching fees in basis points up to 27 days are all positive, and investors with higher risk aversion attitudes are willing to pay much more to avoid any higher risk in constructing the GMVP. Interestingly, the switching fees increase monotonically from 5.8 to the highest 96.17 with the corresponding of 1- and 18-day predictions. Our results provide evidences for significant short-run momentum effects in the conditional covariance matrix of stock and bond returns.

＜Table 3 is inserted about here＞

5. Conclusions

This paper we analyze the comovements between stock and bond returns by modeling the conditional covariance matrix using a new class of HF-based volatility model. The HEAVY system consists of two equations: HEAVY-P and HEAVY-V. Unlike standard GARCH models using LF data, the former focuses on modeling the conditional covariance matrix using the recently accuracy realized measures of volatility. The latter equation aims to provide conditional expectation dynamics of realized covariance matrix, which is essential for producing multi-step predictions in the HEAVY system. By efficiently extracting the informative information from HF data, the predictions from HEAVY exhibit a relative short response time and short-run momentum effects. The models are estimated using data on the S&P 500 and 30-year T-bond futures contracts. To provide insights into the value of using HEAVY predictions, we construct GMVP and compare the performance determined by the conditional covariance matrix forecasts.

Based on the data from 1998 to 2007, the estimation results show that forecasts from HEAVY differ from GARCH in terms of persistence. The HEAVY puts higher weights on past realized covariance matrix allows the forecasts have the potential to response rapidly to the changing markets. Since investors are more concerned about the future than in the past, the forecasting performance of the models over the 4-year data covering the 2008 subprime crisis are compared within a conditional utility-based approach. The superiority of the forecasts from HEAVY over short horizons of up to a week is supported by performing statistical tests. The performance fees of switching from GARCH to HEAVY are all positive over investment horizons of up to one month. While the literature has shown that GARCH models with LF data are useful in modeling stock-bond volatility, our results further find that HF data are valuable in performing volatility-timing-based strategies.

References

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2001. The distribution of realized exchange rate volatility. Journal of the American Statistical Association 96, 42-55.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica 71, 579-625.

Andersen, T.G., Bollerslev, T., Frederiksen, P., Nielsen, M. Ø., 2010. Continuous-time models, realized volatilities, and testable distributional implications for daily stock returns. Journal of Applied Econometrics 25, 233-261.

Baele, L, Bekaert, G., Inghelbrecht, K, 2010. The determinants of stock and bond return comovements. Review of Financial Studies 23, 2374-2428.

Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A., Shephard, N., 2011. Multivariate realized kernels: consistent positive semi-definite estimators of the covariation of equity prices with noise and non-synchronous trading. Journal of Econometrics 162, 149-169.

Barndorff-Nielsen, O.E., Shephard, N., 2002. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. Journal of the Royal Statistical Society Series B 64, 253-280.

Barndorff-Nielsen, O.E., Shephard, N., 2004. Econometric analysis of realized covariation: high frequency covariance, regression and correlation in financial economics. Econometrica 72, 885-925.

Bauwens, L., Laurent, S., Rombouts, J.V.K., 2006. Multivariate GARCH models: a survey. Journal of Applied Econometrics 21, 79-109.

Bollerslev, T., Engle, R.F., Wooldridge, J.M., 1988. A capital-asset pricing model with time-varying covariances. Journal of Political Economy 96, 116-131.

Chaboud, A.P., Chiquoine, B., Hjalmarsson, E., Loretan, M., 2010. Frequency of observation and the estimation of integrated volatility in deep and liquid financial markets. Journal of Empirical Finance 17, 212-240.

Chou, R.Y., Liu, N., 2010. The economic value of volatility timing using a range-based volatility model. Journal of Economic Dynamics & Control 34, 2288-2301.

Connolly, R., Stivers, C., Sun, L., 2005. Stock market uncertainty and the stock-bond return relation. Journal of Financial and Quantitative Analysis 40, 161-194.

De Goeij, P., Marquering W., 2004. Modeling the conditional covariance between stock and bond returns: a multivariate GARCH approach. Journal of Financial Econometrics 2, 531-564.

De Goeij, P., Marquering W., 2009. Stock and bond market interactions with level and asymmetry dynamics: an out-of-sample application. Journal of Empirical Finance 16, 318-329.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naïve diversification: how inefficient is the 1/N portfolio strategy? Review of Financial Studies 22, 1915-1953.

Engle, R.F., 2002. New frontiers for ARCH models. Journal of Applied Econometrics 17, 425-446.

Engle, R.F., Colacito, R., 2006. Testing and valuing dynamic correlations for asset allocation. Journal of Business & Economic Statistics 24, 238-253.

Engle, R.F., Kroner, K.F., 1995. Multivariate simultaneous generalized ARCH. Econometric Theory 11, 122-150.

Fleming, J., Kirby, C., Ostdiek, B., 2001. The economic value of volatility timing. Journal of Financial Economics 67, 473-509.

Fleming, J., Kirby, C., Ostdiek, B., 2003. The economic value of volatility timing using "realized" volatility. Journal of Finance 56, 329-352.

Guidolin, M., Timmermann, A., 2006. An econometric model of nonlinear dynamics in the joint distribution of stock and bond returns. Journal of Applied Econometrics 21, 1-22.

Hansen, P.R., Huang, Z., Shek, H.H., 2012. Realized GARCH: a joint model for returns and realized measures of volatility. Journal of Applied Econometrics 27, 877-906.

Hansen, P.R., Lunde, A., Voev V., 2014. Realized beta GARCH: a multivariate GARCH model with realized measures of volatility. Journal of Applied Econometrics 29, 774-799.

Hautsch N., Kyj, L.M., Malec, P., 2013. Do high-frequency data improve high-dimensional portfolio allocations? Journal of Applied Econometrics, forthcoming.

Jagannathan R., Ma, T., 2003. Risk reduction in large portfolios: why imposing the wrong constraints helps. Journal of Finance 58, 1651-1683.

Noureldin, D., Shephard, N., Sheppard, K., 2012. Multivariate high-frequency-based volatility (HEAVY) models. Journal of Applied Econometrics 27, 907-933.

Patton, A.J., Sheppard, K., 2009. Evaluating volatility and correlation forecasts. In Handbook of Financial Time Series, Andersen T.G., Davis, R.A., Kreiß, J.-P., Mikosch, T. (eds). Springer: Berlin; 801-838.

Shephard, N., Sheppard, K., 2010. Realizing the future: forecasting with high-frequency-based volatility (HEAVY) models. Journal of Applied Econometrics 25, 197-231.

Vuong, Q.H., 1989. Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica, 57, 307-333.

West, K.D., 1996. Asymptotic inference about predictive ability. Econometrica 64, 1067-1084.

West, K.D., 2006. Forecast evaluation. In Handbook of Economic Forecasting, Elliott, G., Granger, C., Timmermann, A. (eds.). North Holland; 99-134.

Wu, C.-C., Liang, S.-S., 2011. The economic value of range-based covariance between stock and bond returns with dynamic copulas. Journal of Empirical Finance 18, 711-717.

Table 1

Summary statistics for stock and bond excess returns

	Close-to-close		Open-to-close	
	S&P 500	30-yr T-bond	S&P 500	30-yr T-bond
Full sample: 1998/01/02~2011/12/30				
Mean	0.0083	0.0101	-0.0162	0.0103
Std. dev.	1.3063	0.6583	0.9469	0.4878
Minimum	-11.7913	-3.1776	-7.5786	-2.8642
Maximum	12.7225	4.0782	5.4979	3.7392
Skewness	-0.0502	-0.1790	-0.2070	-0.2550
Kurtosis	12.4622	4.6266	7.2168	5.6524
Correlation	-0.3291		-0.2983	
Jarque-Bera	13162.92	407.76	2639.03	1072.40
In-sample: 1998/01/02~2007/12/31				
Mean	0.0167	0.0052	-0.0183	0.0070
Std. dev.	1.0914	0.5794	0.8386	0.4390
Minimum	-5.8841	-3.1776	-4.1744	-2.2361
Maximum	5.6663	2.1609	5.4979	1.7129
Skewness	0.0847	-0.3820	0.1097	-0.3546
Kurtosis	5.5232	4.3290	5.5496	4.3006
Correlation	-0.2021		-0.2148	
Jarque-Bera	665.12	244.37	681.04	228.22
Out-of-sample: 2008/01/02~2011/12/30				
Mean	-0.0118	0.0220	-0.0110	0.0183
Std. dev.	1.7189	0.8185	1.1685	0.5895
Minimum	-11.7913	-3.0347	-7.5786	-2.8642
Maximum	12.7225	4.0782	4.6520	3.7392
Skewness	-0.1069	-0.0125	-0.4816	-0.1581
Kurtosis	12.0504	3.9871	7.0979	5.8673
Correlation	-0.4681		-0.4066	
Jarque-Bera	3524.12	41.93	761.99	357.83

Note: The table provides summary statistics for the daily excess returns (in percentage) on the S&P 500 index and the 30-year T-bond for the full and the sub- samples. Critical values for Jarque-Bera statistics at the 5% significance level for T=3528 (full sample), T=2496 (in-sample), and T=1032 (out-of-sample) are 5.98, 5.97, and 5.93, respectively.

Table 2

Estimation results for stock and bond excess returns

	GARCH	HEAVY	
		HEAVP-P	HEAVY-V
μ_1	0.0000	-0.0002	-
	(0.0001)	(0.0002)	
μ_2	0.0001	0.0001	-
	(0.0001)	(0.0001)	
c_{11}	0.0007	0.0013	0.0010
	(0.0002)*	(0.0003)*	(0.0002)*
c_{21}	-0.0000	0.0001	-0.0001
	(0.0001)	(0.0001)	(0.0000)*
c_{22}	0.0004	0.0004	0.0004
	(0.0001)*	(0.0001)*	(0.0001)*
b_{11}	0.9737	0.8994	0.8453
	(0.0075)*	(0.0244)*	(0.0181)*
b_{22}	0.9763	0.9636	0.9246
	(0.0054)*	(0.0071)*	(0.0104)*
a_{11}	0.2121	0.3980	0.5208
	(0.0277)*	(0.0439)*	(0.0264)*
a_{22}	0.1965	0.2648	0.3661
	(0.0212)*	(0.0251)*	(0.0222)*
Log-L	18875.18	18935.22	-
Log-L gains	-	60.04	-
	0.9931	0.9285	0.9890
λ_i	0.9923	0.8666	0.9858
	0.9918	0.8089	0.9723

Note: The table presents parameter estimates with robust standard errors (in parentheses) of GARCH and HEAVY models using data from January 2, 1998 to December 31, 2007 (2496 daily observations), while '*' denotes statistical significance at the 5% level. Index $i=1$ refers to the S&P 500 index and $i=2$ to the 30-year T-bond. Log-L stands for the joint log-likelihood by the model. Log-L gains report the gains of HEAVY-P over GARCH in Log-L. λ_i represents the eigenvalues of the conditional covariance matrix of daily returns and the conditional expectation of daily realized covariance matrix conditioned on the covariance stationary condition.

Table 3

Out-of-sample performance of covariance matrix forecasts

Forecast horizon (h)	$\overline{\sigma^p_{\text{GARCH}}}$	$\overline{\sigma^p_{\text{HEAVY}}}$	VR	DMW	$\triangle_1$	$\triangle_{10}$
1	7.042	6.990	101.984	2.074	0.585	5.856
2	10.112	10.037	102.080	2.097	1.230	12.342
3	12.482	12.386	102.086	2.167	1.856	18.660
4	14.491	14.371	102.211	2.257	2.629	26.474
5	16.263	16.129	102.183	2.172	3.253	32.818
6	17.869	17.722	102.178	2.095	3.905	39.472
7	19.348	19.196	102.106	1.981	4.419	44.741
8	20.730	20.576	102.034	1.874	4.891	49.615
9	22.030	21.876	101.930	1.765	5.233	53.183
10	23.266	23.110	101.875	1.701	5.665	57.674
11	24.444	24.286	101.823	1.637	6.075	61.963
12	25.571	25.408	101.805	1.603	6.572	67.152
13	26.654	26.487	101.774	1.560	7.014	71.797
14	27.698	27.525	101.769	1.535	7.543	77.362
15	28.706	28.530	101.765	1.502	8.079	83.004
16	29.681	29.501	101.770	1.473	8.657	89.107
17	30.627	30.445	101.754	1.432	9.129	94.144
18	31.546	31.370	101.686	1.358	9.308	96.167
19	32.443	32.275	101.584	1.274	9.249	95.733
20	33.317	33.164	101.445	1.163	8.906	92.358
21	34.170	34.033	101.298	1.048	8.422	87.502
22	35.001	34.884	101.142	0.925	7.783	81.009
23	35.816	35.721	100.967	0.787	6.909	72.055
24	36.615	36.546	100.784	0.640	5.856	61.187
25	37.398	37.358	100.584	0.477	4.559	47.727
26	38.166	38.159	100.384	0.314	3.130	32.834
27	38.919	38.947	100.166	0.136	1.410	14.813
28	39.658	39.726	99.929	-0.058	-0.631	-6.639
29	40.384	40.500	99.641	-0.293	-3.291	-34.724
30	41.096	41.255	99.405	-0.485	-5.666	-59.896

Note: The table reports the out-of-sample performance of the GMVP strategies. For each set of GMVP weights according to (9), we report the annualized average realized portfolio volatility ($\overline{\sigma^p_i}$), the variance ratio (VR) computed by $\overline{\sigma^{2,p}_{\text{GARCH}}} / \overline{\sigma^{2,p}_{\text{HEAVY}}}$, the Diebold-Mariano and West (DMW) forecast comparison test, and the average annualized basis point fees ($\triangle_\gamma$) with $\mu^{\text{id}} = 0.05$ from GARCH to HEAVY. The sample period is January 2, 2008 to December 30, 2011.

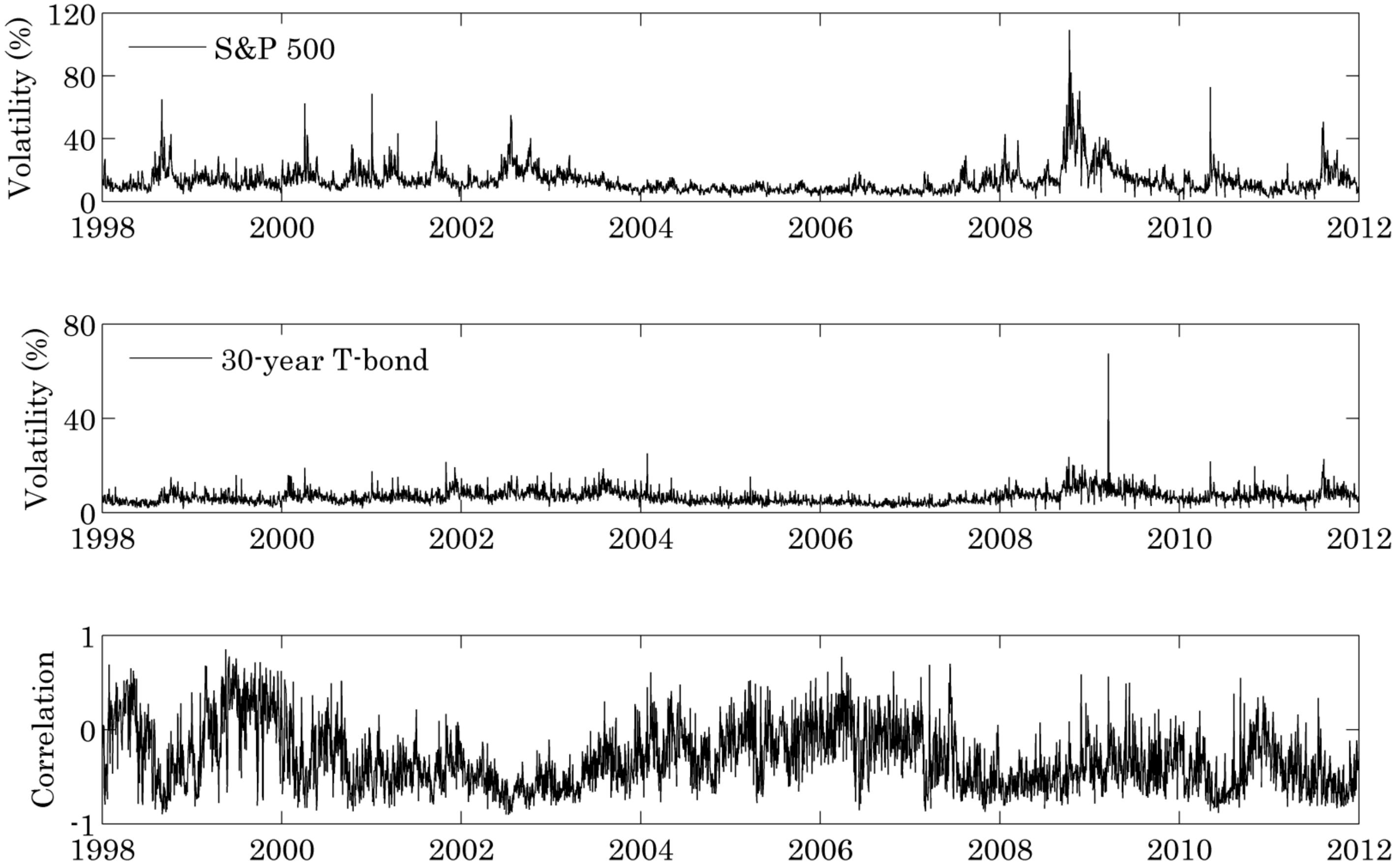

Figure 1 Realized volatility (annualized) and realized correlation on the S&P 500 index and the 30-year T-bond for the period January 2, 1998 and December 30, 2011. The estimates are generated using the multivariate realized kernel of Barndorff-Nielsen et al. (2011).

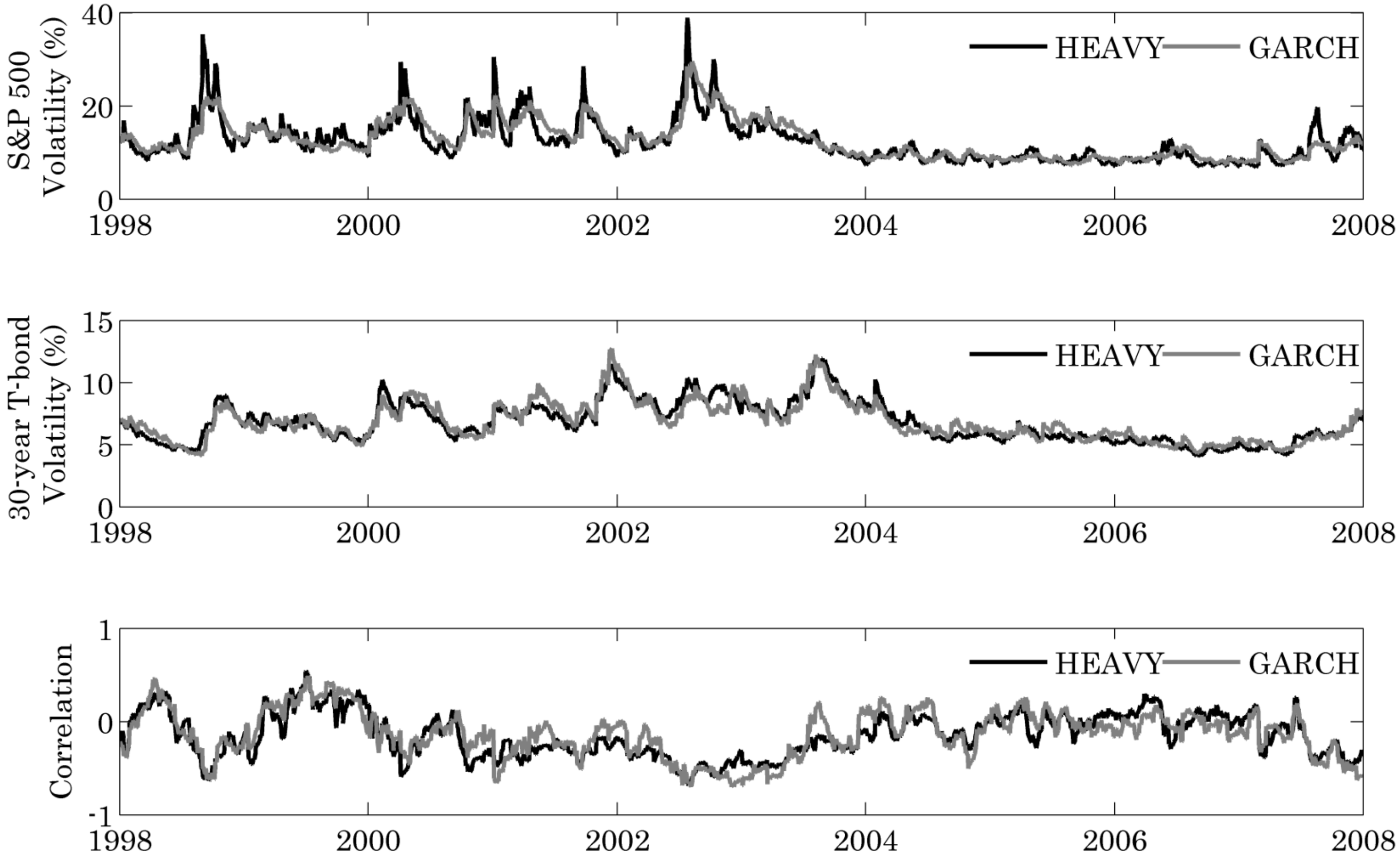

Figure 2 Conditional volatility (annualized) and conditional correlation (implied) on the S&P 500 index and the 30-year T-bond open-to-close excess returns for the period January 2, 1998 and December 31, 2007. The estimates are generated using the HEAVY and the GARCH.